

Slide 1

While you wait...**What is  $35+27$ ?**

- ▶ Use a number line jumping by either 10 or 1 only.

▶ Use a number line jumping any way you want.

Try (246 + 77) or (156-77) if you have time

Slide 2

Mathematical Thinking for Instruction  
Webinar

K-6

### Composing and Decomposing



**BOISE STATE UNIVERSITY**  
CHALLENGE. EMPOWER. INNOVATE.  
CENTER FOR SCHOOL IMPROVEMENT  
& POLICY STUDIES



Slide 3

## Getting Started

- ▶ If you need audio through your phone (can't hear us now):

- ▶ Toll-free: 1 877 568 4108  
▶ Access Code: 222-909-279

- ▶ Technical Information
- ▶ Questions
- ▶ Polls



## Slide 4

### Composing and Decomposing

- ▶ **Building Mathematical Understanding**
  - ▶ Teaching for Understanding
- ▶ **Task**
  - ▶ Investigate task progression for K-6
- ▶ **Closing**
  - ▶ Make connections to Common Core Standards
  - ▶ Building Mathematical Understanding Framework

---

---

---

---

---

---

---

---


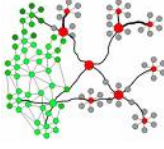
---

---

## Slide 5

### Teaching for Understanding

- **Structural Perspective**
  - Knowledge is structured through **web-like/** hierarchal connections.
  - Mental representation as part of a network of representations.
  - **Claim:** The stronger and the greater number of connections there are in this complex structure, the higher degree of understanding.



©DMT 2008

---

---

---

---

---

---

---

---

---

---

## Slide 6

### Guided Reinvention: Arithmetic

“By thinking and talking about *similarities and differences* between arithmetic procedures, students can **construct relationships** between them. ... the instructional goal is not necessarily to inform one procedure by the other but, rather, to **help students build a coherent mental network** in which all pieces are joined to others with multiple links.”

(Gravemeijer & van Galen, 2003)

©DMT 2008

---

---

---

---

---

---

---

---

---

---

Slide 7

| Grade Level  | Common Core Standard   |
|--------------|--|
| Kindergarten | K.OA.3 <i>Decompose numbers less than or equal to 10 into pairs in more than one way</i>   |
| First        | 1.OA.6 <i>Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums</i>   |
| Second       | 2.NBT.7 <i>Add and subtract within 1000, using concrete models or drawings and <u>strategies based on place value</u>...</i> Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, and ones and ones; and sometimes it is necessary to <u>compose or decompose</u> tens or hundreds.   |
| Third        | 3.OA.5 <i>Apply properties of operations as strategies to multiply and divide.</i> ... $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$ , then $15 \times 2 = 30$ , or by $3 \times 2 = 6$ , then $6 \times 5 = 30$ . Associate property of multiplication: $5 \times 3 \times 2 = 5 \times (3 \times 2) = 16$ , one can find $8 \times 7 \times 4 = (8 \times 7) \times 4 = 56 = 56$ . (Distributive property) |
| Fourth       | 4.NBT.5 <i>Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using <u>strategies based on place value and the properties of operations</u>. Illustrate and explain the calculation by using equations, rectangular arrays, and area models.</i>  |
| Fifth        | 5.NBT.7 <i>Add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and <u>strategies based on place value, properties of operations, and the relationship between addition and subtraction</u>; relate the strategy to a written method and explain the reasoning used.</i>   |

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---


---

---

Slide 8

**Task 1**

Sam the Sheep Dog will herd all of the sheep into 2 pens. How many ways can Sam herd the sheep into the pens?

A black and white sheepdog stands in the center, facing slightly left. It is flanked by two identical green squares with dashed orange borders, representing pens. In front of the dog and pens are seven cartoon sheep, each with a grey body, white face, and a red nose. The sheep are arranged in two rows: three in the back and four in the front. The entire scene is set against a light blue background with a white dotted line at the bottom.

---

---

---

---

---

---

---

---

---

---

---

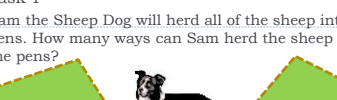
---

---

Slide 9

**Task 1**

Sam the Sheep Dog will herd all of the sheep into 2 pens. How many ways can Sam herd the sheep into the pens?

A black and white sheepdog stands in the center, facing slightly left. It is flanked by two identical green squares with dashed orange borders. In front of the dog are seven cartoon sheep, each with a grey body, white face, and a long, curved, light-colored tail. The sheep are arranged in two rows: three in the back and four in the front. The entire scene is set against a light blue background with a white border.

---

---

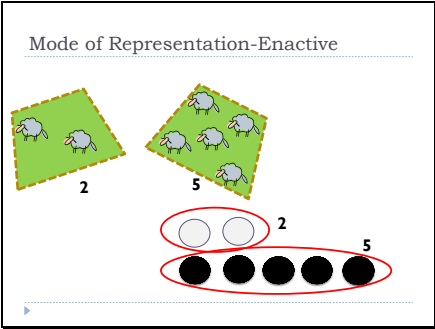
---

---

---

---

Slide 10



---

---

---

---

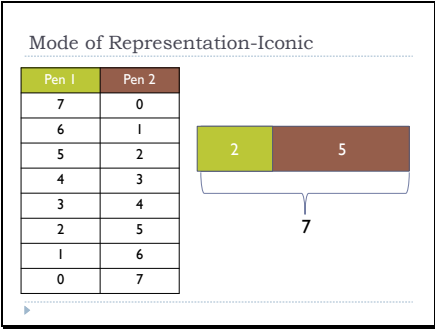
---

---

---

---

Slide 11



---

---

---

---

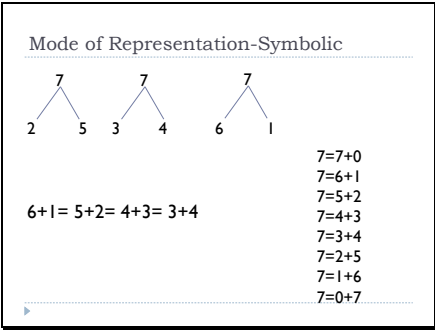
---

---

---

---

Slide 12



---

---

---

---

---

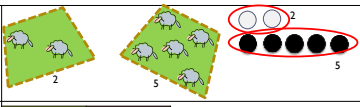
---

---

---


Slide 13

Enactive

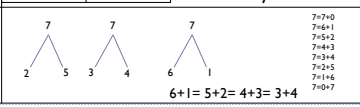


Iconic

| Pen 1 | Pen 2 |
|-------|-------|
| 7     | 0     |
| 6     | 1     |
| 5     | 2     |
| 4     | 3     |



Symbolic



$6+1=5+2=4+3=3+4$

$7+7=0$   
 $7+6=1$   
 $7+5=2$   
 $7+4=3$   
 $7+3=4$   
 $7+2=5$   
 $7+1=6$   
 $7+0=7$

---

---

---

---

---

---

---


---

---

---

Slide 14

Task 2-



If you buy one toy, how much money will you have left?  
How do you know you are correct? Show your thinking.

---

---

---

---

---

---

---

---


---

---

Slide 15

Task 2-

How would you solve this using enactive, iconic and symbolic models?  
How does composing and decomposing help students to solve this task?



If you buy one toy, how much money will you have left?  
How do you know you are correct? Show your thinking.

---

---

---

---

---

---

---

---

---

---

Slide 16

Mode of Representation-Enactive

12¢

18¢

---

---

---

---

---

---

---

---

---

---

Slide 17

One Student's Strategy

A student's strategy:  
"I started at 30 and took away 10. That left me with 20. I took away 5 more to get to 15. I took away 3 and that left me with 12."

---

---

---

---

---

---

---

---

---

---

Slide 18

Mode of Representation-Iconic

A student's strategy:  
"I started at 30 and took away 10. That left me with 20. I took away 5 more to get to 15. I took away 3 and that left me with 12."

12¢

18¢

30¢

---

---

---

---

---

---

---

---

---

---

Slide 19

Mode of Representation-Symbolic

30

-18

20

-8

12

$18+2 \rightarrow 20$

$+10 \rightarrow 30$

$2+10 \rightarrow 12$

---

---

---

---

---

---

---

---

---

---

Slide 20

|          |   |
|----------|---|
| Enactive |   |
| Iconic   |   |
| Symbolic | <div><div><div>30</div><div>-18</div><div>20</div><div>-8</div><div>12</div></div><div><div><math>18+2 \rightarrow 20</math></div><div><math>+10 \rightarrow 30</math></div><div><math>2+10 \rightarrow 12</math></div></div></div> |

---

---

---

---

---

---

---

---

---

---

Slide 21

Task 3

› The Flicks theater has a room with 17 rows of seats. Each row consists of 8 seats. How many people is the room able to seat?

---

---

---

---

---

---

---

---

---


---

Slide 22

Task 3

► The Flicks theater has a room with 17 rows of seats. Each row consists of 8 seats. How many people is the room able to seat?

Solve with an iconic and symbolic representation and consider how a student might solve enactively



---

---

---

---

---

---

---

---

Slide 23

Mode of representation: Enactive

Students might model the rows of theatre seats with tiles or counters. The picture shows how students could represent 5 rows of 8 seats. This group could then be repeated and/or expanded to get 17 rows



---

---

---

---

---

---

---

---

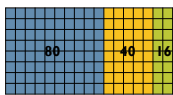
Slide 24

Mode of representation: Iconic

► Area model on graph paper and open area model. Both decomposing 17 into 10+5+2.

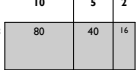
1052

8804016



1052

8804016



---

---

---

---

---

---

---

---



Slide 25

► Area model on graph paper and open area model. Both decomposing 17 into 10+5+2.

1052

8

80

40

16

1052

8

80

40

16

$8 \times 17 = 8 \times (10 + 5 + 2) = (8 \times 10) + (8 \times 5) + (8 \times 2)$

►

---

---

---

---

---

---

---

---

---

---

Slide 26

Mode of representation: Symbolic

Partial products

$10 \times 8 = 80$   
 $7 \times 8 = 56$   
 $80 + 56 = 136$

Traditional algorithm

$$\begin{array}{r} 17 \\ \times 8 \\ \hline 136 \end{array}$$

►

---

---

---

---

---

---

---

---

---

---

Slide 27

|          |  |   |
|----------|--|---|
| Enactive |  | Students might model the rows of theatre seats with tiles or counters. The picture shows how students could represent 5 rows of 8 seats. This group could then be repeated and/or expanded to get 17 rows |
|          |  | Area model on graph paper and open area model. Both decomposing 17 into 10+5+2.   |
|          | <div><div><math display="block">\begin{array}{r} 17 \\ \times 8 \\ \hline 136 \end{array}</math></div><div>Traditional algorithm</div></div> <div><div><math display="block">\begin{array}{r} 10 \times 8 = 80 \\ 7 \times 8 = 56 \\ 80 + 56 = 136 \end{array}</math></div><div>Partial products</div></div> |   |

►

---

---

---

---

---

---

---

---

---

---

Slide 30



Solve with a ratio table and symbolic representation



|          |   |                       |                     |                  |  |
|----------|---|-----------------------|---------------------|------------------|--|
| Symbolic | $\begin{array}{r} 1.7 \\ \times 8 \\ \hline 13.6 \end{array}$ | Traditional algorithm | $1 \times 8 = 8$    |                  |  |
|          |   |                       | $.7 \times 8 = 5.6$ | Partial products |  |
|          |   |                       | $8 + 5.6 = 13.6$    |                  |  |

Slide 31

|        |     |     |     |      |    |      |
|--------|-----|-----|-----|------|----|------|
| Jugs   | 1   | 2   | 4   | 8    | 10 | 15   |
| liters | 1.7 | 3.4 | 6.8 | 13.6 | 17 | 25.5 |

Doubling strategy to get to 8, then adding 2 and 8 to get 10 and finally adding 1, 4 and 10 to get 15.

|        |     |     |    |     |      |      |
|--------|-----|-----|----|-----|------|------|
| Jugs   | 1   | 2   | 10 | 5   | 15   | 8    |
| liters | 1.7 | 3.4 | 17 | 8.5 | 25.5 | 13.6 |

Double to get to 2, multiply by 10, divide in half, add 10 and 5, subtract 2 from 10

---

---

---

---

---

---

---


---

---

---

Slide 32

This could be done with base ten blocks or pictures. Depending on prior experience with decimal numbers, students may not need to enact these type of problems.

$1.7 + 1.7 + 1.7 + \dots$   
  
3.4      3.4

A ratio table could be used to solve both problems. Repeated addition/subtraction could also be used.

|        |     |     |     |      |    |      |
|--------|-----|-----|-----|------|----|------|
| Jugs   | 1   | 2   | 4   | 8    | 10 | 15   |
| liters | 1.7 | 3.4 | 6.8 | 13.6 | 17 | 25.5 |

Doubling strategy to get to 8, then multiply by 10 and finally adding 1, 4 and 10 to get 15.

|        |     |     |    |     |      |      |
|--------|-----|-----|----|-----|------|------|
| Jugs   | 1   | 2   | 10 | 5   | 15   | 8    |
| liters | 1.7 | 3.4 | 17 | 8.5 | 25.5 | 13.6 |

Double to get to 2, multiply by 10, divide in half, add 10 and 5, subtract 2 from 10

$\begin{array}{r} 1.7 \\ \times 8 \\ \hline 13.6 \end{array}$  Traditional algorithm

$1 \times 8 = 8$   
 $.7 \times 8 = 5.6$   
 $8 + 5.6 = 13.6$  Partial products

---

---

---

---

---

---

---

---

---

---

Slide 33

Connections to place value

$17 \times 8$

$10 \times 8 = 80$

$7 \times 8 = 56$

$80 + 56 = 136$

$1.7 \times 8$

$1 \times 8 = 8$

$.7 \times 8 = 5.6$

$8 + 5.6 = 13.6$

---

---

---

---

---

---

---

---

---

---

## Slide 34

---

---

---

---

---

---

Slide 35

---

---

---

---

---

---

Slide 36

---

---

---

---

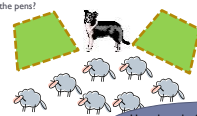
---

---

Slide 37

**Kinder-First**

Sam the Sheep Dog will herd all of the sheep into 2 pens. How many ways can Sam herd the sheep into the pens?



**First/Second Grade**


30¢ 6¢ 23¢ 15¢ 18¢

If you buy one toy, how much money will you have left?  
How do you know you are correct?

How does the first problem connect to the other three? Where do we see examples of decomposing?

**Third/Fourth Grade**

The Flicks theater has a room with 17 rows of seats. Each row consists of 8 seats. How many people is the room able to seat?



**Fifth and Sixth Grade**

An apple farmer fills each jug with 1.7 liters of cider. Find out how many liters the following number of jugs hold:  
2, 8, 10, 15

If a barrel can hold 289 liters of cider. How many jugs can the farmer fill?

---

---

---

---

---

---

---

---

---

---

Slide 38

| Grade Level | Common Core Standard  |
|-------------|---|
| Kinder      | K.OA.3. <b>Decompose</b> numbers less than or equal to 10 into pairs in more than one way   |
| First       | 1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; <b>making ten</b> ; <b>decomposing a number</b> leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums  |
| Second      | 2.NBT.7 Add and subtract within 1000, using concrete models or drawings and <b>strategies based on place value</b> ... Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to <b>compose or decompose</b> tens or hundreds.  |
| Third       | 3.OA.5 Apply properties of operations as strategies to multiply and divide... $3 \times 5 = 2$ can be found by $3 \times 5 = 15$ , then $15 \times 2 = 30$ , or by $5 \times 2 = 10$ , then $3 \times 10 = 30$ . ( <b>Associative property of multiplication</b> ) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$ , one can find $8 \times 7$ as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ . ( <b>Distributive property</b> ) |
| Fourth      | 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using <b>strategies based on place value and the properties of operations</b> . Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.  |
| Fifth       | 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and <b>strategies based on place value, properties of operations</b> , and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.  |

---

---

---

---

---

---

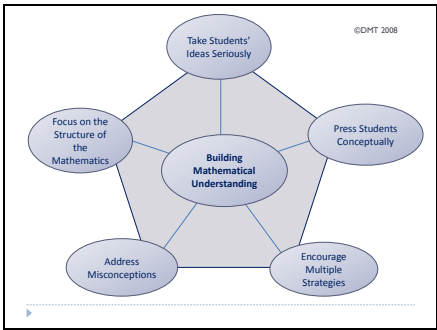
---

---

---

---

Slide 39



---

---

---

---

---

---

---

---

---

---

Slide 40

## 1 Credit Opportunity

- ▶ **Duration:** Accumulate 15 hours of webinar training, live or archived. Additional webinars will be developed and offered during the Fall of 2012. The credit will be earned the semester the 15 hours is completed.
- ▶ **Registration:** Upon completion of the 15 hours, a participant will register with BSU for the one professional education credit.
- ▶ **Documentation:** Completion of a brief webinar summary and reflection for each webinar is required.
- ▶ **Cost:** \$65. The one professional education credit earned for completion of a price of \$65 does not count towards the three credits earned with completion of the MTI course. The webinars are follow-up support after completion of the MTI course.
- ▶ **Information:** <http://www.sde.idaho.gov/site/math/mti.htm>
- ▶ **Questions:** Nichole Hall nhall@sde.idaho.gov

---

---

---

---

---

---

## Slide 41

Thank you for attending the webinar!

- ▶ Questions
- ▶ Contact Information
  - ▶ Sarah Reynolds [sarahreynolds@boisestate.edu](mailto:sarahreynolds@boisestate.edu)
  - ▶ Jackie Ismail [jacquelynismail@boisestate.edu](mailto:jacquelynismail@boisestate.edu)
- ▶ DMT Website- <http://dmf.boisestate.edu>
- ▶ Follow Up Opportunities:  
<http://www.tinyurl.com/mtifollowup>



---

---

---

---

---

---